

# Avogadro's Number: Is The World Granular?

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Consider the experience of taking a walk in the woods. As the gravel crunches beneath your feet, the wind blows through your hair and the sunlight plays hide-and-seek from between the leaves overhead, you focus on the path in front of you and pick out the details that draw your interest. What looked like a patch of shimmering yellow from a distance, for example, now resolves itself into a rich experience: an aspen tree, with its leaves dancing in the wind. You focus your attention on specific details, and the details of the aspen leaves enrich the overall impression of the tree.

In relating to such details, you may, if you choose, pay attention to numerical patterns. Is there a pattern to how many leaves unfold and fan out from a twig? Do the veins on the leaves divide out in a particular way? As you observe, you can hold the original totality of the tree in your mind and look for specific patterns. You can start counting the number of leaves that appear on twigs that developed during one growing season. There are rarely only one or two leaves on such a twig and usually not more than ten, but everything in between. You do not find a pattern. But when you look at the veins in many leaves you will notice a pattern: five veins usually branch out from the base of a leaf blade. This is not always the case, sometimes there are only three veins. The numerical pattern belongs to the aspen tree and there are variations; it is not fixed or rigid. What we think of as number in this context does not lead an abstract existence. It is intertwined with all the other feature of the tree and can show itself when we study the tree from the perspective of numerical pattern.



Consider a different experience: you take a walk in a desert full of sand dunes. Richness of life is here replaced by the dryness and heat of the air and sand that surrounds us. What could appear as a smooth mound resolves itself into myriad tiny grains of sand that you can scoop up with your hands. You wouldn't think of trying to count them and no numerical patterns show themselves.

The ever-changing shapes of the dune seems completely indifferent to the number of grains that are added to it or lost from it. It appears as if numbers do not matter to the sand at all. One grain sits next to another.



When you pick up a cluster of quartz crystals and observe them in the sunlight, you can experience solidity in multiple ways — in the fixed shape, in its hardness, in the rigid features, and most importantly, in the inherent six-sidedness of the planar sides that make up the columnar sections. Here you have a specific relation to number, one that incorporates the geometric relationships of planes, lines and points in the sides, edges, and tips of the crystal structure.

We can literally grasp and hold steady in our minds the fixed numerical relationships in a specific quartz crystal, just as we can hold it in our hands. This is quite different from the form of a sand dune that changes as soon as we touch it. Numbers that we can attribute to this crystalline experience are neither mostly indifferent, as they are in sand, nor are they burgeoning with life and variety, as they are on a tree. Here we mostly experience numerical relationships that, in a way, appear to have come to rest.



These three experiences highlight different ways of engaging with the world numerically. In each case, we focus on certain details — the veins in a leaf, the grains of sand, or the facets of a crystal — and different kinds of numerical relationships appear. In the case of plants, they retain an organic quality; they usually are not fix in any rigid sense. Any statement we could make about the appearance of numerical relationships in the plant world always has an intrinsic “wiggle room”

that is related to natural variation. A species of plants that normally has five petals in each of its flowers may surprise us with a four- or six-petal flower. These surprises are part and parcel of the way numbers exist in the living world. This does not mean these relationships are arbitrary. But they are imbued with the dynamic quality of the living world. They come into being and pass away in growth and decay.

When we see the five-pointed star at the center of an apple we have sliced in half, or the six petals and six stamens of a lily flower, we sense the connection of number to whole. The number of grains in a handful of sand or a sand dune does not have this quality of inherent connection to the whole. There is a disconnect between number and the whole in this kind of granular phenomenon. This disconnect also extends to our own participation, since we are not drawn into any numerical relations; they are not essential when observing piles of sand. Surely we can find, for example, an average number of grains in a small volume and extrapolate to a large volume. This method of relating to numbers is used in statistical analysis.

In the case of the crystals, where the numerical and geometrical relationships are stable, our capacity to work with these relationships takes on a mathematical quality. Calculations derived from the study of crystals have historically led to the development of many fields in mathematics, as crystal observation and mathematics work hand-in-hand. For example, this interrelatedness led the famed 17<sup>th</sup> century astronomer, Johannes Kepler, to declare that:

*Where there is matter, there is geometry.<sup>i</sup>*

Confidence in the stability of crystal structures was so deep-rooted that it led Kepler to create a picture of the entire solar system that contained crystalline structures embedded one inside the other, whose relative sizes gave the size of the orbits of the planets.

Kepler was also deeply interested in the musical quality of number, which is distinct from the spatially-oriented relationship to number that I have discussed above. He saw the solar system as a harmonious arrangement in addition to being a spatial geometric arrangement. He went so far as to name his fundamental work in astronomy, which gave birth to all of modern astronomy, the *Harmonices Mundi* (The Harmony of the World).

The experience of music can be something deeply inward. It moves our feelings and stems from a reality beyond what we can see with the eyes or touch with our hands. The experience of music seems to go beyond just the ears and penetrates the entire human organism, as any lover of music will attest. It is intimately woven with our living in time. We perceive rhythms and can find numerical patterns in music. As Kepler's contemporary, the philosopher-scientist Gottfried Leibniz, stated:

*Music is a hidden arithmetic exercise of the soul, which does not know that it is counting.<sup>ii</sup>*

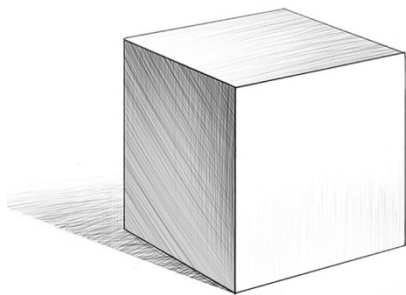
We are largely unaware of numerical relations in our experience in music, while in crystals geometrical patterns are perceived clearly and consciously. In the living world numerical relationships show a flexible character. In a handful of sand, in a sand dune, number is arbitrary.

The landscape of numerical relations, as it is drawn out of our deeper feelings into the clear light of day, shows its own distinctive features. These features are essential in determining the way we approach measurement of physical properties in the world. It is important to keep the distinct quality of number in each given instance clearly in mind. We need to distinguish whether we are speaking of eight beats in music, or spatially, of five petals, seven grains of sand, or a six-sided crystal.

The same numbers are embedded in contexts that reveal additional qualities such as inwardness, fixedness, flexibility or disconnectedness. We can train ourselves to see these different qualities. Before we engage in the task of measurement, we should take care to notice what quality of number relationships we are dealing with.

### From Number to Measurement

As we have seen, number, as a conscious and clear idea, is brought about through our alert participation and engagement with the world around us. But how, exactly, does this process take place? Let us observe this using a simple example. When we have a cube in front of us, there are various ways of engaging with it. We can pay attention to the totality, through which we could say that we have **one** cube in front of us. But we could also pay attention to the *idea* of planes — through which **six** planes are seen to be present. There are other features that we can subsequently focus on, such as edges and corners. And so we can have:



**One** cube  
**Six** faces (planes)  
**Twelve** edges (lines)  
**Eight** corners (points)

In other words, when we look at the world through the lens of geometry and attribute number to specific features, it is our *attention* that begins the process of assigning the corresponding number. This shows the fallacy of treating numbers as free-floating “things” that can simply “belong” to objects, since they arise only through an active process of human focus and mental conception. Each of the numbers we associated with the cube has a particular quality out of which the number was born without grasping the quality of a line, we could never count 12 lines in a cube. *Without penetrating the quality itself, the quantity has no foundation.* At the same time, *all* the numbers that arise from this engagement carry the *fixed* quality of the crystal.

Just as we can focus on a cube, and seek out the numerical relationships that arise there, what happens when we direct our gaze to specific aspects of the human being directly? For instance, when we focus on the breathing process, a definite temporal rhythm arises, which we could count. When we focus on the heartbeat or pulse, another rhythm arises, which we could also count. Breathing and heartbeat also bear an intimate relationship to our sense of time and music, making them natural choices to examine this question. What happens, however, when we move from *counting* to *comparing*? We could then focus on the relationship between the two, which is most often found to be four to one: four heartbeats per breath. Through this comparison, a living

*measurement* arises, as it did historically before the era of standardization. This number of measurement – four heartbeats per breath – lies close to our experience, and is yet dynamic and living in the way described with respect to plants earlier. Age, inherited qualities, emotional state, mental state etc. can all change this relationship, but on the whole, we have an organic unity in this measurement. As we compare, we move from living numbers to living measurement.

We could also focus on the length of a foot and relate that to our vertical height. For an average adult, the height is generally between five to six times the length of the foot, giving rise to another measurement relationship. It seems so natural for us to say that someone is six feet tall, that we almost forget that we are comparing the length of a foot to the height of the person when we say that. When physical relationships arise out of human dimensions in this way, such as the span of a hand, the length of the arm, the length of a stride, the weight of a person, or the time of the heartbeat, they give rise to a *science of measurement* where we can still relate to the world in a human way. They are not yet “set in stone” and standardized, as one person’s foot differs in length from another’s, just as the size of a leaf varies. Still, the living quality of humanness is retained when such numbers arise, and as we have seen, the quality out of which a number arises is its very foundation. Such measurements can be seen as the physical perceptible analogue of a worldview and philosophy that was common in older cultures: “Man is the measure of all things.”

As our sciences progressed, standardization increased, and as a result measurements gradually lost their organic quality and became more fixed, like a crystal. Today, one “foot” has lost its connection to the anatomical foot and is instead defined precisely as 0.3048 meters, and a meter in turn is defined in an equally precise way. For nearly 170 years, through the 19<sup>th</sup> and 20<sup>th</sup> centuries, a platinum bar (or its alloy) was used as a standard for how long a “meter” was. This was the closest we could get to inserting the “fixed crystal” quality into our measurements. Yet, in spite of the push towards standardization, the units of measure that we have become accustomed to use, such as feet, meters, seconds, pounds or kilograms, still carry an echo of a human quality. A meter is generally the length of the human stride, a second is very close to the time of a heartbeat, and we can all usually manage to carry a kilogram or pound in one hand without too much difficulty. These units, which are a product of a centuries-long science of measurement, still remind us of our own engagement in the process of measurement.

Nonetheless, the scientific revolution that began in the Renaissance led to a fundamental change, and these changes took place in two different directions, almost simultaneously.

### **Measurement: The Twofold Change**

Two inventions set the stage for the fundamental change that would influence the science of measurement in the modern era: the telescope and the microscope. Products of the interest in scientific discovery in the 16<sup>th</sup> and 17<sup>th</sup> centuries, the telescope and microscope opened up worlds that were not discernible by the naked eye. The names “microscope” and “telescope” arose within a decade of each other<sup>iii</sup>, and this change of scale opened up worlds that were very hard to engage in a direct sense. Galileo (1564-1612), who discovered features of the Sun and Jupiter’s moons through his telescope, also literally turned his instrument around to look at insects:

*I heard the author himself [Galileo] narrate to that noble philosopher, the most excellent Signor Cremonius, various things worthy to be known, and among others how he perfectly distinguished with the telescope [perspicillum] the organs of motion and of sense in the smaller animals, but especially in a certain insect which has each eye covered by a rather thick membrane that is nevertheless perforated with seven holes like the iron visor of a warrior, thus affording a passage to the images of visible things.<sup>iv</sup>  
[W]ith this tube I have seen flies which look as big as lambs.<sup>v</sup>*

What does it mean when we say that there are thousands of bacteria in a drop of water? What does it mean when we say that there are thousands of stars in every direction that we look? These two worlds — the very small and the very large — were fascinating, but also created a sense of detachment and meaninglessness for the human being. The discovery of the inconceivable extremes appearing as it did in the immediate aftermath of the Copernican revolution that jolted man's conception of the Earth as a place at rest appeared to pull the rug out from underneath us as far as our relationship to numerical magnitudes went.

This upheaval is also evidenced by the fact that the word “million” only entered the language at the end of the 14<sup>th</sup> century<sup>vi</sup>, while the word billion was formed in the 16<sup>th</sup> century and entered common usage in English only in the late 17<sup>th</sup> century<sup>vii</sup>. Until that time, we did not even have the need to think such numbers<sup>viii</sup>. Just as sense perception was extended beyond ordinary human capacities using the microscope and telescope, the size of numerical magnitudes was also stretched beyond human comprehension. It is particularly important to note this shift in engagement with the surrounding world. It leads to a relationship to the world of numbers that is very similar to the disengaged way in which we view the grains of sand. I would suggest the following: **The disengagement that exists with respect to the number of grains of sand on a beach is now transferred to the world of the very large and the world of the very small.**

Perhaps it is not a coincidence that the “grain of sand” metaphor arises in descriptions of both the denizens of the microscopic world:

*... incredibly small; nay, so small, in my sight, that I judged that even if 100 of these very wee animals lay stretched out one against another, they could not reach the length of a grain of course sand; and if this be true, then ten hundred thousand of these living creatures could scarce equal the bulk of a coarse grain of sand.<sup>ix</sup>*

And the world of the stars:

*Those worlds in space are as countless as all the grains of sand on all the beaches of the earth.<sup>x</sup>  
And then, the Earth being small, mankind will migrate into space, and will cross the airless Saharas which separate planet from planet and sun from sun.<sup>xi</sup>*

[Note: As far back as the 3<sup>rd</sup> century B.C., Archimedes had already referred to his work that dealt with very large numbers brought about by the heliocentric world picture, as the *The Sand Reckoner*.]

We must pay attention to this transition from numbers that have an embodied human quality — that relate to what we can see and touch — to those which can only be grasped in thought. It is no longer possible to relate to these numbers the way we did to the leaves in a tree, or to the sides of a crystal. Both fixedness and livingness retreat from our experience and we are left with a handful of sand.

Further shifts occurred in our relationship to both extremes over the course of the 18<sup>th</sup> and 19<sup>th</sup> centuries. Boundaries were pushed in both directions — towards the ever smaller in physics and chemistry, and towards the ever larger in astronomy.

### **Boundaries of Magnitudes in Light and Matter**

When astronomers and physicists turned their gaze from the stars that they viewed as points of light to the phenomena of light itself, new questions arose. Discussions began regarding variations in the observations through the telescope. One of these concerned the variations in the eclipse times of Jupiter's moons, another, variations in the positions of stars through the year. Attention shifted towards the possible causes of these variations. One possibility that was explored was the idea of “speed of light.” Regardless of the reasons scientists had for approaching the phenomenon of light from this perspective, as a focus of research, the determination of the speed of light is an elegant culmination of the astronomical path of exploration.

The notion of a “speed of light” is not easily fathomable and escapes direct human experience. Galileo was a pioneer in this research as he attempted to detect whether one can see how quickly the shining of a light is perceived at a distance. He had two people stand on distant hills, and had one of them open up a lantern and the other one open up *his* lantern in response to when the first lantern becomes visible. The idea was that if there was a time lag between these openings, one can calculate the speed as the ratio of distance to time. This attempt to detect a speed for light did not go anywhere at the time: the effect was instantaneous for human observation.

It was not until the middle of the 19<sup>th</sup> century that some scientists were able to directly detect what could be called a speed of light. It was designed by analogy to the experiment detecting the speed of sound, where one determine the time it takes to hear an echo and work out the speed in that way. By using reflection of light from rapidly rotating mirrors, and creating a “light echo” (details to be discussed separately, see reference<sup>xiii</sup>), it was determined that light-speed was an astonishingly high (approximately) 300,000,000 meters per second! Speed of that magnitude staggers the human imagination, as our everyday speed of walking is about a couple of meters per second. Add to that the observations that light is not an “object” one can grasp with one's hand or whose movement one can observed directly with the eyes, that radiates in all directions, and we have a phenomenon that is very difficult to come to terms with in terms of speed. When we look into the starry sky with thousands of stars, each of which are sources of light, a numerical grasp of this expansive experience stretches the mind and challenges our conceptions.

At the other extreme, we have the microscopic structure of matter. Determining the internal structure of matter has been of great interest to scientists. In the search for the ultimate constituents of matter beyond what could be observed under an ordinary microscope, physicists and chemists

utilized the hypotheses of *atoms and molecules*. Initial experiments (described here<sup>xiii</sup>) focused more on the fact that substances combined chemically in specific weight ratios, allowing us to sense a numerical pattern in their composition. These numerical relationships, which were in the form of simple ratios such as 2:15 and 4:7, enabled scientists to relate to them in a qualitatively different way than the ideas of atoms, as the relationship of these ratios to simple musical ratios was clearly noted. In arranging substances based on the ratios of specific weights, some chemists went so far as to call upon musical analogies like the “Law of Octaves”<sup>xiv</sup> to describe the pattern. In addition, studies of crystal structure opened up a wide range of geometrical patterns to look at and study in the mineral world, which provided a numerical order from a different direction. The number was attributed to *atoms*, which were imagined to fit together in different ways thus giving rise to the properties of matter.

The atomic perspective was mainly guided by hypotheses, since there was no direct evidence of such particles to either the naked eye or the microscopes of the day. Different observations of the properties of matter were then “explained” in terms of the assumed behaviour of the hypothesized particles. Take for instance the changes that occur when a solid melts when heated. It was *imagined* that little atoms of the solid, which were neatly packed together before, are now moving around. If the same liquid was now heated and evaporated, it was *imagined* that the tiny particles broke free and flew all over the place. The observational reality was that the solid, liquid, and gas each had a particular set of properties, and one could compare specific aspects of these properties — e.g. one could take the ratio of the volume occupied by a weight of gas versus the volume occupied by the same weight of liquid of the same substance. (A litre of water, for instance, expands to about 1700 litres of steam upon boiling.) However, the imagery of atoms was made to permeate this picture. *In a sense, all of matter was being seen as different variations of a bunch of sand: either all packed together, or moving about, or floating about.* We could call this approach that carries the intrinsic quality of disconnectedness the “sand hypothesis.”

If we stay true to this imagery, the question arises: How do we *count* these particles? If I carry a fistful of charcoal, how many particles am I, in my imagination, attributing to it? If I have a couple of litres of oxygen, how many particles am I attributing to it? Or more precisely, if I pick the lightest substance, hydrogen, and ask how many particles a specific volume of it contains, what is the answer?

This number, which is at the root of the sandy picture of the world, is called the Avogadro number.

Amedeo Avogadro (1776 -1856) hypothesized that a specific number of particles constituted the weight of a certain volume of gas, *irrespective of the type of gas*. This hypothetical property — same number of particles per volume in all gases — allowed this number to be anchored in physics and chemistry of his time. The specific amount of gas (about 22.4 litres volume of it) was called a mole, and its weight was called the molar weight. A mole of a substance was supposed to contain an Avogadro number of particles in it. Hence, about 12 grams of charcoal is supposed to contain the Avogadro number of particles.

The Avogadro number has a value of about  $6.02 \times 10^{23}$ . *That is approximately six followed by 23 zeros* — once again, an absurdly high number that is hard to find our way to, in terms of direct experience. It has been substantiated by a wide variety of experiments, which we will examine in



the next section. Along with the speed of light, which escapes our experience in its fleetingness and lack of substantiality, the Avogadro number also escapes us in terms of the sheer number of particles that one is supposed to imagine. In addition, the Avogadro number appears perfectly suited to remove all sense of harmony from our relationship to matter. While the chemical arrangement of substances, established by the Periodic Table of elements, appeals to our sense of harmony and progression, as Element Number 1 progresses in steps all the way up to about Element Number 118, one does not normally know *what* to do with  $6.02 \times 10^{23}$  number of particles of any kind.

Returning to the two boundaries — the boundary of insubstantial light and the boundary of matter — numbers appear that stretch our sense of proportion way more than the telescope or microscope did in terms of size. The granularity that becomes imagined in the world due to the sheer size of Avogadro's number in particular generates a detachment from the whirlwind of atoms that we are imagined to live in. This creates an inner relationship to the world as if one is in the middle of a sandstorm, with no particular orientation. When that feeling of number, that is already present in our experience of the multitude of stars, is now brought into our sensing of the world around us, or of our own body, our entire world becomes a meaningless variation of pixelated formations, with no sense of solidity or life at the basis of it all. *We become world-estranged, which leads our thoughts and actions in a direction devoid of inner connectedness and meaning.* This quality of disengagement prepares the way for a lack of care towards the world around us, and has a deep effect on our inner relationship to this world. No matter how much we might profess to care for the natural world, if the “sand hypothesis” has taken root in our heart of hearts in addition to our minds, detachment and alienation from nature are inevitable. As a consequence, exploitation and uncontrolled use of natural “resources” accelerate.

In order to see if there is another way to relate to nature and the world of matter, we have to examine more carefully the phenomena that appear in the determination of the Avogadro number, and see if they lead in a different direction than where we are led by the assumption of atoms and molecules. Is there a way in which we could relate, once again, to these boundary magnitudes? In order to examine that, it is important to look into the origins of Avogadro's number in greater detail.

### **Phenomena Leading to the Avogadro Number**

As scientists began to delve more deeply into the fine structure of matter, their investigations began to lead them toward the existence of a particular number that could be reached through a variety of disciplines. The various paths were summarized by Jean Perrin (1870 – 1942), who then declared:

*I think it impossible that a mind, free from all preconception, can reflect upon the extreme diversity of the phenomena which thus converge to the same result, without experiencing a very strong impression, and I think that it will henceforth be difficult to defend by rational arguments a hostile attitude to molecular hypotheses, which, one after another, carry conviction...<sup>xv</sup>*

What are the several threads that led to the determination of this number? These experiments can broadly be classified in this way:

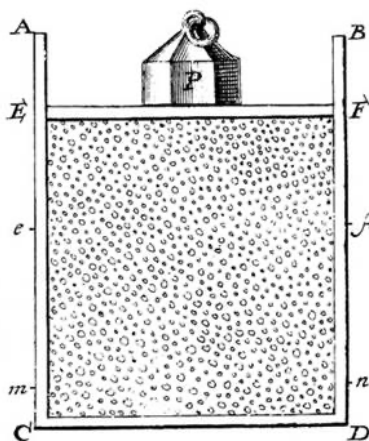
1. Experiments with gases

2. Experiments with Liquids
3. Experiments with solids
4. Experiments with electricity
5. Experiments with radioactivity

Each of these will be examined to tease out the actual phenomena behind the calculation of the Avogadro number.

## Experiments with Gases

The discovery of Avogadro's number with respect to gases took a convoluted route. In the first place, physicist Daniel Bernoulli (1700 – 1782) assumed that gases are made of tiny particles, and tried to attribute the pressure of a gas to these particles interacting with the container walls.<sup>xvi</sup> In other words, he pictured myriads of self-propelling ping pong balls knocking on a wall to “explain” the pressure of a gas<sup>xvii</sup>:



*... let the cavity contain very minute corpuscles, which are driven hither and thither with a very rapid motion; so that these corpuscles, when they strike against the piston and sustain it by their repeated impacts, form an elastic fluid which will expand of itself if the weight is removed or diminished...*

The actual experimental measurements were of pressures and volumes of gases at different temperatures. Based on his observations, Bernoulli theorized a set of “minute corpuscles” or mini-ping-pong balls moving very fast. These elastic ping-pong balls, as well as their number, are drawn entirely from imagination and are assumed to strike the bottom surface to balance the weight. Under this imagination, in order to sustain the weight at the top, the speed of these balls was calculated to be more than a thousand feet per second for a specific temperature. If we pause a moment here, we will observe the tell-tale sign of an abstraction that is far removed from experience — a disengagement from the reality in front of us. The number of balls is treated very similar to the sand particles discussed earlier, and the sand hypothesis inserts itself, with the modification that these “sand particles” are always flying about and bounce a lot. Our *perception*,

however is only that of the pressure of the gas as it compresses or expands — such as the pressure changes observed while pumping air into a bicycle tire.

Bernoulli's theory was soon called into question: the theoretically calculated velocity of the balls did not match life experience. The chemist C.H.D. Buys-Ballot (1870-1890) famously complained:

*If I were sitting at one end of a long dining room and a butler brought in dinner at the other end, it would be some moments before I could smell what I was about to eat. If atoms were flying at hundreds of meters per second, I should smell the dinner as soon as I saw it.<sup>xviii</sup>*

This discrepancy did not deter the scientists. Instead of abandoning the entire ping-pong ball idea, it was decided to “fit” the result, by adding another mental construct to it: a “wrapping factor” called “mean-free-path”<sup>xix</sup>. This factor is used to bring the theory into agreement with the observations, i.e. to “explain” why gases spread and diffuse about *ten thousand times* slower than the earlier calculations assuming elastic balls. There was also another issue: how much volume do these ping-pong balls occupy if they are all compressed together? It was decided to use the ratio of a volume of a gas to the volume of the same substance as a liquid in order to determine how the balls are compacted using this “compression factor.” Once again, taking the example of water, about 1700 litres of steam compresses upon cooling to give just a litre of water, hence the “compression factor” is about 1700. The “wrapping factor” due to slow diffusion when combined with the “compressing factor” diminishes the original velocity estimate by a factor of tens of millions (ten thousand times a few thousand, i.e.  $10^7$ ). This value is cubed, since the volume of a gas is three dimensional, in order to obtain and estimate the “atomic volume.” This means a volume reduction of more than  $10^{21}$  takes place. And it gives an approximate value for the Avogadro number<sup>xx</sup>.

As we can see, the key measurements were that of the spreading tendency of the gas, as well as the amount it compresses into a liquid volume. The speed modified through these observations is compared to the speed expected of a ball hitting the wall to generate the pressure, and the discrepancy is attributed to the behaviour of gas versus that of a solid. In trying to re-imagine this process as a number of ping-pong balls moving about, several compression factors had to be applied, which ultimately provided a vastly diminished volume from which the required number was estimated.

The dynamic of a gas expanding in all three dimensions is pictured as the dynamic of ping-pong balls moving in a straight line. As a result of this speculative approach, a small volume is derived, smaller than the volume of the gas we started out with by a factor roughly equal to Avogadro's number. Had we not had the experimental results of diffusion and compression upon cooling, we could not have obtained Avogadro's number from this situation.

## Experiments with Liquids

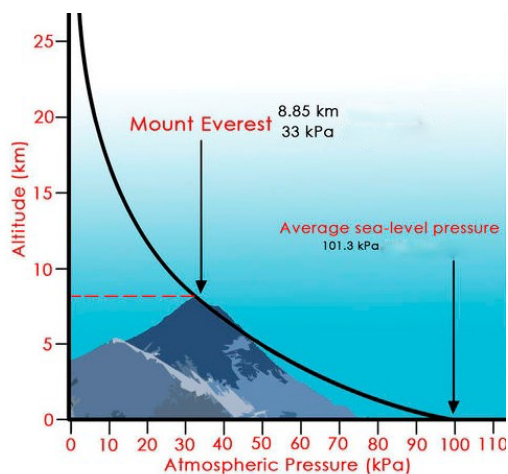
The fundamental shift into an atomistic view of the world was inspired by the ceaseless movement of insoluble particles in a liquid – called *Brownian motion*. When particles are suspended in a

liquid, they show a ceaseless activity that does not seem to “settle down” at all. As declared by the pioneer of these experiments, Jean Perrin<sup>xxi</sup>:

*What is really strange and **new** in the Brownian movement is, precisely, that it never stops.*

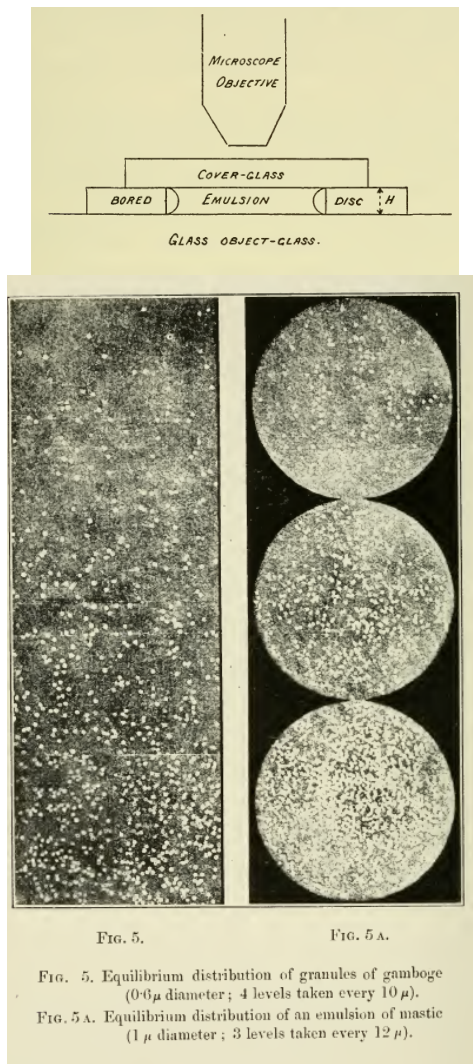
That particles suspended in a fluid continue to fluctuate for months and even years is truly a remarkable observation. The phenomenon seems to indicate an essential quality of a liquid, as several liquids and several suspensions showed similar behavior. However, once again, an intrinsic mysterious property of the liquid — a capacity for continuous fluctuating movement independent of surrounding illumination or other mechanical vibrations — was attributed to “bombardments” of the surrounding molecules, or more ping-pong balls.

Perrin used various methods to support this idea<sup>xxii</sup>. One method involved observing the changing concentration of the suspended granules caused by changes in height. He did this by creating a suspension of granules of gamboge (a type of rubber) and mastic (an exudation of the bark of the mastic tree) in water. He then observed the solution under a microscope, and determined the variation of concentration with height, which took the form of an exponential decay. This decay is seen with gases in the atmosphere as well – air density at a height is much lower than air density at the earth’s surface, which can be verified by measuring air pressure as we rise up in a hot air balloon. It follows a curve like this:



Altitude vs. Air pressure, showing the exponential decay with descent

The emulsion itself was located in the center of an O-ring (labeled BORED DISC in the diagram below) with a cover glass on top, creating a cylindrical volume of liquid. He then took snapshots of the distribution of particles in the emulsion, by focusing the microscope on different heights of the emulsion. In the diagram below (Fig. 5) there are 4 rectangular snapshots of the gamboge’s emulsion and 3 circular snapshots of the mastic (Fig. 5A).



Concentration gradient seen under the microscope

Based on these observations, Perrin declared<sup>xxiii</sup> that:

*It shows clearly that the concentration of the granules of a uniform emulsion decreases in an exponential manner as a function of the height **in the same way** as the barometric pressure does as a function of the altitude.*

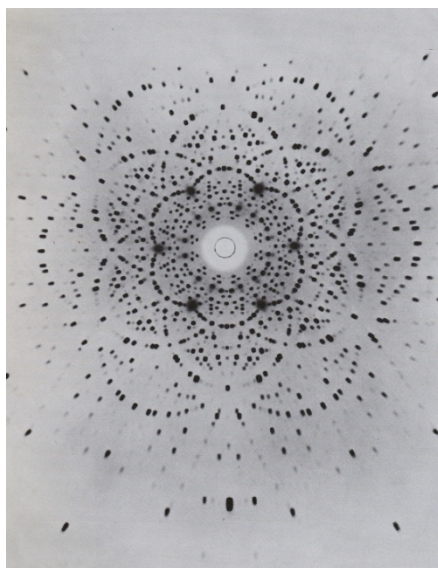
This was an astonishing result, as gases and liquids are so different in our experience. The exponential drop in pressure with height observed for gases had been theoretically attributed to discrete molecules. This picture for gases is the same as that of discrete ping-pong balls explained in the previous section on “experiments with gases.” Seeing the same exponential change in these emulsions, Perrin asserted that in liquids too the molecules exist and bombard the particles in the suspension, and are hence the cause of the Brownian movement. And most importantly, just as the Avogadro number was attributed to gases, it was found that it required an Avogadro number of suspended particles to match up with the observed decay rate of pressure.

This was also a noteworthy result, as the change in concentration was estimated from observations of easily visible particulate matter. What made it possible for the concentration gradient to match already available equations for gas concentrations? And why was it that Avogadro's number arose for the number of granules to fit the equation? Perrin's answer was: it was because molecules were *assumed* in deriving the equations, and this provided experimental proof of the correctness of the assumption. We will examine the validity of this conclusion in a later section on the quality of the Avogadro number.

Another method that has been used with liquids is to see how a particular weight of a liquid spreads into a thin film. If we imagine that the thinnest possible film contains a layer only one ping-pong-ball thick, the thickness of the film can be calculated in a straightforward way by dividing the volume of the liquid by the area of the film formed. For example, a drop of liquid such as oil is added to water, generating a thin film of oil on the water that is spread out to the maximum extent it can spread. Dividing the volume of oil by the area of the oil film gives us the thickness of the layer, assuming no oil is lost or dissolved in the pouring. This thickness is assumed to be the depth of the presumed molecular layer and is usually less than a few nanometers. Assuming a cube with the same side as this thickness, the volume can be derived, and compared to the total volume of the thin film.<sup>xxiv</sup> The ratio gives the Avogadro number, and the value derived through this method matches other derivations.

### Experiments with Solids:

The experimental derivation of the Avogadro number from the solid state is much more straightforward, since it contains a minimum of assumptions. X-Ray radiation (radiation that is capable of making a gas electrically conductive), when passed through a solid crystal (e.g. beryl), generates a pattern on a sensitive screen:



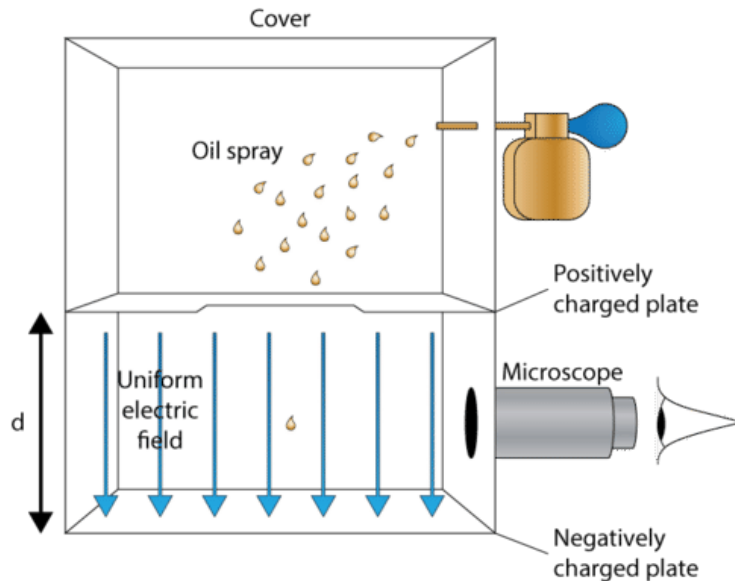
Such beautiful patterns show the intricate inner geometry of the crystal. If the crystal were not present, we would have only seen a circular form at the center of the screen. The change from a

blob to the pattern we see above is attributed to the crystal structure; regularities or periodicities *within* a crystal affect the radiation passing through. Depending on the placement and separation of the dots in the pattern, it is possible to derive the periodicity, or spacing, of layers of the solid crystal face. The spacing can be obtained in all three perpendicular directions in space, and the product of the spacing sizes gives the volume of a single unit of the crystal lattice. The size of this volume in relation to the size of the crystal as a whole approximates Avogadro's number. In other words, the interaction between the linearly directed X-Rays and the solid volume of the crystal structure provides a rhythmic pattern, whose size indicates a relationship between the volume of a specific weight of crystal and the volume of the single unit that is approximately Avogadro's number.

The difference between the derivations in case of liquids or gases on the one hand and solids on the other hand is stark. In case of liquids and gases, the Avogadro number is quite foreign to experience, as we do not sense any discontinuity in these states that can "clump" into molecules. As a result, the theoretical methods rely heavily on statistics of fluctuations – statistics being the favoured method when the relationship to the numbers is lost. In deriving the results based on "imaginary sand," we have to make a series of assumptions to "reduce" the dynamics of the process into one linear dimension, either by detecting the linear spread of gases (diffusion) or by observing how suspended particles vibrate and move in a liquid. It is only the theoretical physical interrelationships that allow an estimate of the Avogadro number. For solids, a linear aspect is achieved by directing linear beams of X-Rays at a resting volume.

## **Experiments with Electricity**

Electricity provides us with perhaps the simplest method of obtaining an estimate of the Avogadro number. The value of the electrical charge has been directly determined experimentally only in a series of steps, mainly by observing the effect of electrical fields on water and oil droplets. Droplets of oil are sprayed into an electrical field created by oppositely charged metal plates. For our purposes, we can consider a field simply as a region where electrical activity is made manifest in a controlled manner. As the droplets are sprayed, they fall through that region, and are affected in their fall by the electric field. Some droplets reverse and move upwards, others simply slow down, and yet others stop and hang in the air. By observing the field required to balance a droplet of a particular size it is possible to detect the charge on the drop, and it turns out to be discrete. For example, one droplet can have values of 1, 2, or 3 times a unit of charge, but not 0.34 units. Although the size of this unit of charge is extremely small, its magnitude is well known. By dividing the charge required to completely electrolyse a specific weight of a substance to this unit charge, the Avogadro number is easily derived, as was done by JJ Thomson<sup>xxv</sup> and Millikan<sup>xxvi</sup>. In this case, one does not even require an externally imposed linear process such as X-rays or suspended particle movements to get to the numerical relationship — the nature of electrical discharge is in itself linear (as evidenced by the linear sparks that fly between our hand and the door-knob in winter). This allows what is probably the simplest determination of the Avogadro number.



## Experiments with Radioactivity

Similar to electricity, radioactivity involves the generation of beams of positively charged radiations, also called alpha radiations, from radioactive materials. One of the first materials widely used to study radioactivity was radium and this was what Rutherford and Geiger<sup>xxvii</sup> used in their experiments. They used the fact that radiation is emitted in bursts, which can be counted from the number of electrical discharges that the radiation stimulates (via “throws” of an electrometer needle). The second important fact was that alpha radiation has the property of converting to helium gas over time. The steps in their process for obtaining the Avogadro’s number were:

1. Observe how many discharges can be detected from a small mass of radium over a specific time.
2. Calculate the rate of discharge per second.
3. Observe how much helium gas is collected from the same weight of radium per second.
4. Since all the radiation is assumed to transform into helium gas, extrapolate to the number of discharges it would take to generate 22.4 litres of helium, which is the molar volume for all gases.
5. The number of discharges estimated for that volume of helium gives us the Avogadro’s number.

By counting these for a specific set of conditions, and extrapolating this for all directions of emission from the radium, they estimated this to be  $3.4 \times 10^{10}$  incidences per second. Their detection table looked like this:



	Number of throws.	Magnitude of successive throws in scale divisions.
1st minute .....	4	11, 12, 10, 11
2nd " .....	3	10, 11, 8
3rd " .....	5	10, 9, 13, 8, 12
4th " .....	4	18,* 8, 12
5th " .....	3	10, 6, 10
6th " .....	4	9, 10, 12, 11
7th " .....	2	10, 11
8th " .....	3	11, 13, 8
9th " .....	3	8, 20*
10th " .....	4	8, 12, 14, 6
Average per minute = 3.5		Average throw = 10 divisions.

They repeated this experiment with other radioactive materials such as uranium, thorium, and actinium. They also compared the “throws” of the electrometer, or the number of electric discharges, with the scintillations (bright flashes) on a screen made of zinc sulphide. The two were correlated up to 96%-99%, showing that the counting from two different methods gave almost the same result:

I. Calculated number of $\alpha$ -particles per minute.	II. Observed number of scintillations per minute.	III. Ratio of observed to calculated number.
39	31	0.80
38	40	1.29
34	29	0.85
32	31	0.97
31	32	1.03
28	27	0.96
27	28	1.04
25	21	0.84
23	25	1.09
21	21	1.00
Total number = 294		Average = 0.99

Calculated number of $\alpha$ -particles per minute.	Observed number of scintillations per minute.	Ratio of observed to calculated number.
36	31	0.86
34	30	0.88
31	31	1.00
30	29	0.97
27	29	1.07
Total number = 150		Average = 0.96

Finally, Boulton and Rutherford<sup>xxviii</sup> carefully measured the amount of helium gas generated through a certain quantity of alpha radiation, completing the last link in the chain of experiments<sup>xxix</sup>. Measuring the weight of the accumulated helium gas and relating it to the number of discharges provided another way of obtaining the Avogadro’s number. In other words, the number of “throws” or scintillations for one mole of radium (226 g) was calculated to be equal to about  $6 \times 10^{23}$ .

This method is probably the best experiment for highlighting the subtleties in the manifestation of the Avogadro number because the dichotomy of continuous and discrete are brought out so clearly in the experiment. The electrical discharges and flashes of light triggered by the alpha radiation are clearly discrete and can be counted. Helium gas shows no such discreteness or “clumpiness”. Like all gases, it is uniform and continuous, occupying the entire space available to it. Yet we have the discontinuous observations on one side, and a continuous gas on other, and the relationship between the two is established by Avogadro’s number.

Another set of experiments that we have not examined here is the origin of Avogadro’s number through heat radiation, also called *black-body radiation*. As the end result of decades of

experiments on thermal radiation and attempts to express it experimentally, Max Planck finally hit upon the right formula to express the distribution of heat radiation from different objects. That formula contained an expression for the Avogadro number, which could be experimentally determined, and the value was once again close to the other values obtained for this number.

### Continuity, Discontinuity, and the Avogadro Number

Possibly the greatest impact on the view of molecular reality was the collection by Perrin of the various methods of obtaining Avogadro's number in his work of 1909. Upon seeing all the different methods by which the same number is derived, one is left with the strong impression that this number definitely points to a real property of all matter. Around this time, even the staunch critics of the atomic perspective yielded to this view, for example<sup>xxx</sup>:

*I am now convinced that we have recently become possessed of experimental evidence of the discrete or grained nature of matter, which the atomic hypothesis sought in vain for hundreds and thousands of years. The isolation and counting of gaseous ions, on the one hand, which have crowned with success the long and brilliant researches of J.J. Thomson, and, on the other, agreement of the Brownian movement with the requirements of the kinetic hypothesis, established by many investigators and most conclusively by J. Perrin, justify the most cautious scientist in now speaking of the experimental proof of the atomic nature of matter. The atomic hypothesis is thus raised to the position of a scientifically well-founded theory, and can claim a place in a text-book intended for use as an introduction to the present state of our knowledge of General Chemistry.*

A recently published work goes into great detail about what was *actually* observed by Perrin in his experiments<sup>xxxii</sup>. The authors analyzed the logical structure of determinations of Avogadro's number ( $N_0$ ):

*A place to start is with the elements of logical structure that all of the determinations of Avogadro's number had in common. Each case involved a theory-mediated measurement of some primary quantity **from which the value of  $N_0$  was then inferred as a further step.***<sup>xxxii</sup>

Hence, the number itself is never counted, which is not surprising considering its magnitude. But, did Perrin not perceive the continuous movement of suspended gamboge granules? Was this observation of Brownian motion evidence of the movements in an underlying "molecular reality" made up of small sand-like particles? Their conclusion is:

*We have ... shown in detail how Perrin's theory-mediated measurements did not presuppose classical kinetic theory, much less a hard elastic sphere model of atoms and molecules.*<sup>xxxiii</sup>

In other words, the measurements were actually independent of theory, and did not *require* the sand hypothesis. It is also asserted that we should not, by a process of transference, simply attribute

the movements we see in suspended granules to an invisible oscillatory movement of the atoms and molecules, no matter how tempted we are to do so:

*We should not wait ... to point out how **misleading** the suggestion was that Brownian motion is continuous with molecular motion, and hence that Brownian motion can be thought of as **molecular motion writ large and its granules as outsized molecules**.*<sup>xxxiv</sup>

Yet, there is *something* that is being measured, which seems to remain remarkably consistent across different experiments. What is that “something”? Smith and Seth point out (in slightly convoluted language) that:

*... [N]either the “reality of molecules” nor the molecular-kinetic theory of heat has to be presupposed if the evidence provided by the converging measures of Avogadro’s number is construed as a form of same-effect-same-cause reasoning — specifically as same-magnitude-same-quantity-being-measured reasoning. Consider the three least controversial measures that were yielding, to within reasonable precision, the same magnitudes: the number of unit charges, that is, electrons, required to yield a mole of hydrogen, or any other monovalent element, in electrolysis; the number of granules in Brownian motion required to match the total energy at any given temperature of a mole of gas (or liquid solution); and the number of  $\alpha$ - particles required to form a mole of helium. Insofar as all three yield a numerical value characteristic of a mole, the proposed inference plus inductive generalization yields the conclusion of the existence of some distinctive quantity— **a certain number of units of something**— characteristic of any mole of any substance. Suppose we call this quantity **m-units**, with the three measures taken to be indirect measures of it.*<sup>xxxv</sup>

*In particular, this last step to m-units being a count of **individual discrete items** involves a reach beyond the available evidence. Nothing in the evidence we have examined requires a mole to consist of  $(64 \pm 5) \times 10^{22}$  individuals with sufficient integrity to persist in and of themselves over time.*<sup>xxxvi</sup>

This last statement is probably the most significant admission by modern science regarding the caution necessary when proceeding from an observed discontinuity to the notion of “independent atoms and molecules that persist in and of themselves over time.” For example, consider the “throws” of an electrometer that is detecting discrete radioactivity. When we collect the results of this radioactive emission in a container, we get a certain volume of helium gas. What we measured going *into* the container was discontinuous. What we see *in* the container after the fact is a continuous volume of gas. Can we therefore conclude that the gas consists of these discontinuous “somethings” floating around? Why must discontinuity be given greater weight than continuity? Is it valid to assume that these discontinuities remain in the same form and carry over into the gas as a whole? Does the discontinuity observed in one place *have* to persist in the same way in another place? The same argument applies to ionized gas. When gas is ionized through X-Rays or electrical plates or any other way, a discontinuity can be seen between the gas and droplets sprayed into it. Does this mean that the gas itself “consists” of these discontinuous entities when it is not charged, and when it is not interacting via charge with droplets?

A simple example illustrates the problem. If we allow water to drop into a cup, we can easily count the number of drops going into it. Does this mean that our cup of water now has these drops persisting *as drops* inside it, and we should expect to find spherical drops intact in there? However, adding up the volumes of these drops will accurately give the total volume of water in the cup. If we were to drop grains of sand one at a time into a cup, we *can* find these grains persisting as grains. Hence, the attempt to find discrete elements in a continuous body is guided by the quality of number we have taken to be our guideline. In cases of liquids and gases, we see a totality which is continuous and hard to enumerate, which is why there were several assumptions involved in obtaining the Avogadro's number from experiments with liquids and gases – assumptions that brought them as close to the sand hypothesis as possible.



These are the hurdles of extrapolation that we have to take great care not to overlook. What we can say with certainty is that experiments reveal discontinuity *under certain conditions*, and that this discontinuity seems to lead to a specific number for a specific mass of a substance. **Avogadro's number is a measure of specific discontinuities that appear in matter under certain conditions, without necessarily specifying what *form* these discontinuities take.**

A good example that justifies a cautionary approach is another work done<sup>xxxvii</sup> with regard to Brownian motion, showing a completely different picture for the observed movements:

*An implication ... is that the solvent and the colloid are in partial mutual stable equilibrium, that is, they satisfy the conditions of temperature and pressure equalities but not the conditions of total potential equalities. As a result, both the constituents of the solvent and the colloid exert infinitely large "driving forces" (total potential differences) on the **pliable interface between the two parts, and try to interpenetrate each other** as they would have done if the colloid were soluble by the solvent. However, **such an interpretation is impossible, and the only effect is a continuous in time modification of the pliable shape of the interface**, a modification that does not affect the entropy, the energy, the volume, and the amounts of constituents of either the solvent or the colloid and therefore the temperature, the pressure, and the total potentials of the composite of these two systems. Said differently, it is not the motions of the solvent and the colloid that cause the observed*

*movements but the infinitely large differences in total potentials that change the shape of the interface and appear to the observer as motions.*

Leaving aside the question of forces and potentials, the idea raised in this description is fascinating: Brownian motion is not simply a “bombardment” of invisible molecules, but the result of a continuous change of shape of the interface between the liquid and the granule, *leading* to an observed movement. Since we can observe that, except for the upper surface, liquids always seek to take the shape of a container, this idea of the origin of Brownian motion in the shape-seeking property of liquids has a much better grounding in phenomena than hypothetical “things” that bombard the granules. This would shed an entirely new light on the supposed discontinuity being measured via the Avogadro number.

When we seek to better grasp the form of this discontinuity, certain patterns emerge. From one direction, the Avogadro number involves a process of *bringing the observed phenomena into a linear relationship*, except in cases where the phenomena involve processes that are already linear in nature (electricity, radioactivity, X-Rays). In the case of liquids and gases, linearization is achieved by converting the capacity to spread in two or three dimensions into just one dimension (by studying diffusion or linear vibrations). Actual distances travelled by the gas or liquid, or suspensions of particles in a liquid, form the data observed in these experiments.

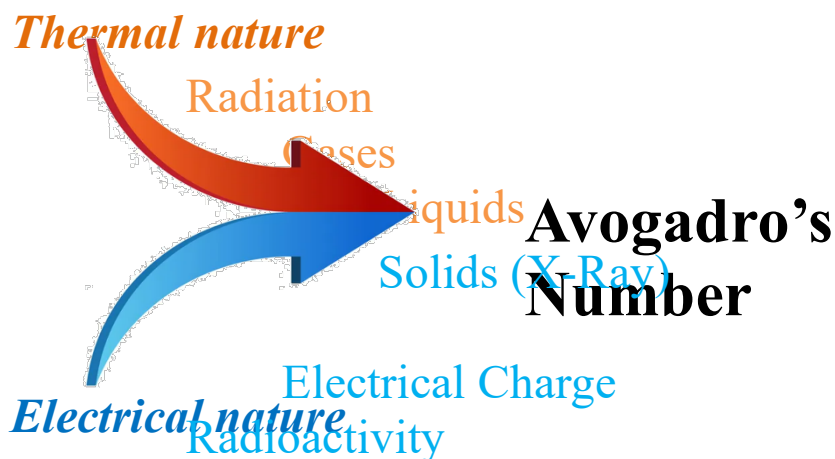
This linearizing process ends up revealing discontinuities in certain cases, which *when expressed numerically lead to the Avogadro’s number*. When linear processes are involved, either in the form of electric discharges or in the form of radiation (X-Rays), the discontinuities show themselves in a straightforward way, yielding a clue to the origin of the Avogadro number: ***it points us towards discontinuities of an electrical nature in matter***. This is not the first time in the history of science that electricity has been involved in discrete phenomena. Chemical combinations have historically been a clear example of specific ratios of weights being involved, and this was verified via electrolysis for hundreds of compounds.

If we focus on continuous processes in radiation, gases and liquids, almost all determinations of the Avogadro number involve the right side of the gas equation:

$$PV = RT$$

where P is pressure, V is volume, and T is temperature, while R is an experimentally observed constant of proportionality. Thermal phenomena lie at the basis of this equation. In particular, the expression  $RT/N_0$  (where  $N_0$  is the Avogadro number) is applicable to diverse continuous phenomena – viscosity of gases, diffusion of substances, Brownian movement, ion mobility, radiation. The Avogadro number is inferred from that expression. In thermodynamics, the ratio  $R/N_0$  is called the Boltzmann constant  $k$ , and is used throughout the subject. ***This points us towards an origin of the Avogadro number in heat phenomena***.

There are, therefore, two sides to the appearance of Avogadro’s number in phenomena. From one side, we have its origin in continuous phenomena, closely tied with heat or temperature in radiation, gases and liquids. On the other hand, we have its origin in discontinuous phenomena, such as the discrete pattern seen when electrifying X-Rays are passed through solids, electric discharges or scintillation seen as result of alpha emission, and changes in electric charge observed in droplets. This can be summarized thus:



### Quality of the Avogadro Number

We have seen the Avogadro number arising in different ways in experiments. The question arises: Are there specific qualitative aspects of the Avogadro number itself that we can recognize in the different experimental contexts? Is it only the “grained nature of matter” (sand hypothesis), or something else? This is the fundamental question we must ask when we seek the quality of Avogadro’s number.

In terms of the descriptions of the quality of numbers in section 1, we can say that historically, the sand hypothesis has predominated with respect to Avogadro’s number. All of matter is seen as a collection of sand and for an unknown reason the Avogadro number of “sand particles” (atoms) happens to constitute this standard quantity of matter. This is especially true when the number is calculated from electric charges or radioactive flashes. However, there is also an element of fixedness that becomes apparent as we scan the large variety of experiments that give rise to this number. This is especially apparent when we observe crystalline structures and obtain the Avogadro number from X-Ray analysis. The consistency of the Avogadro number between different varieties of substance seems to indicate its fixed quality.

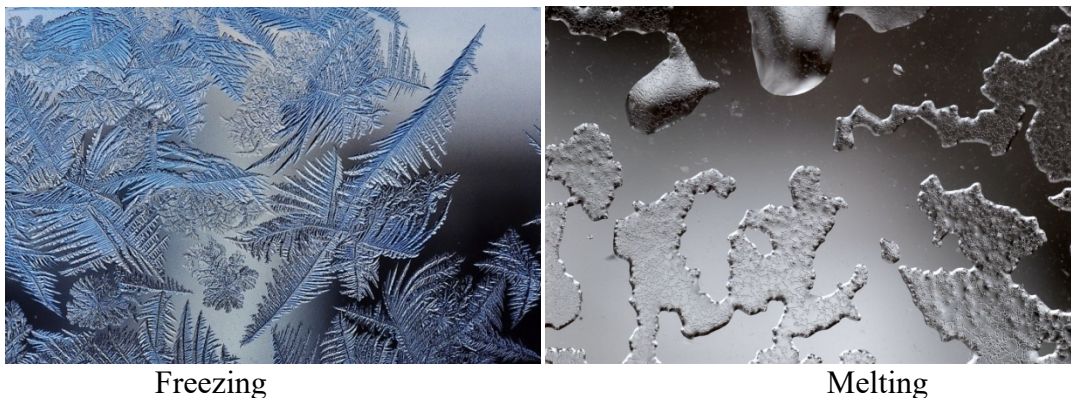
The different phenomena through which the Avogadro number is derived are all, as far as I am aware, rooted in the inorganic realm. It is hard to imagine a living quality, or variation, in the Avogadro number itself. However, one cannot rule this out, since we have no way of telling if there is a range of values within which the Avogadro number varies over time. For the most part, the quality of the Avogadro number seems to be fixed, part of the crystalline nature that does not vary with time.

We now come, however, to an intriguing aspect of the Avogadro number, where it straddles the boundary between continuous and discontinuous phenomena. This is where we have to remember the statement referred to earlier:

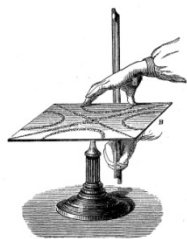
*Insofar as all three yield a numerical value characteristic of a mole, [it] yields the conclusion of the existence of some distinctive quantity — a certain number of units of something — characteristic of any mole of any substance.*

So we have a certain number that is sometimes estimated as a whole (as is the case with most thermal phenomena including liquids and gases) and at other times estimated through extrapolation of the counting process (as is the case with diffraction patterns, scintillations and electric discharges). In other words, the number has an *undifferentiated quality* in one direction and a *differentiated quality* in the other direction. In our earlier comparison, of water droplets versus sand, we examined how these two qualities can be observed in liquid water and solid sand. What the Avogadro number points us towards, however, is a *transformation* between the undifferentiated and the differentiated. It is as if in some phenomena, the Avogadro number appears as an undifferentiated whole, and in others, as a differentiated process. The earlier example of helium gas obtained from alpha radiation discharges is a case in point, where the what was first a discontinuous, differentiated sequence now appears as a continuous whole. When we have estimations of charge from ionized gases, such as gases electrified by passing X-rays through them<sup>xxxviii</sup>, we have a route to the Avogadro number from the opposite direction: from the undifferentiated (gas) to the differentiated (ionized gas).

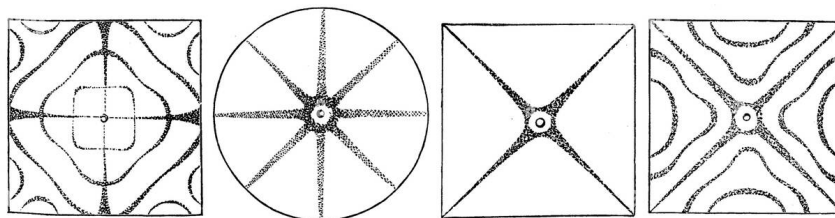
The transformation between the undifferentiated and differentiated occurs in many phenomena in nature, for instance when differentiated snowflake or ice crystals form from water, and when the very same crystals melt back into water in the sun. If we pay particular attention to the interplay between differentiated and undifferentiated states, we have a portal through which we can glimpse the mobile quality of this number:



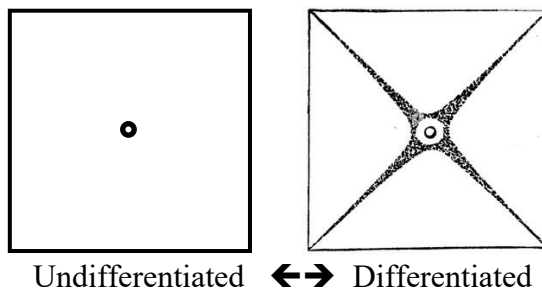
It also seen in the phenomena of the Chladni plate — where a metal plate, fixed at its center with sand or salt sprinkled uniformly on it, is rubbed with a violin's bow. This was studied in greater detail in different solids and liquids in the field of cymatics in the mid-20<sup>th</sup> century, but for our purposes the crucial point is the *arising* of a differentiated pattern:



CHLADNI  
FIGURES



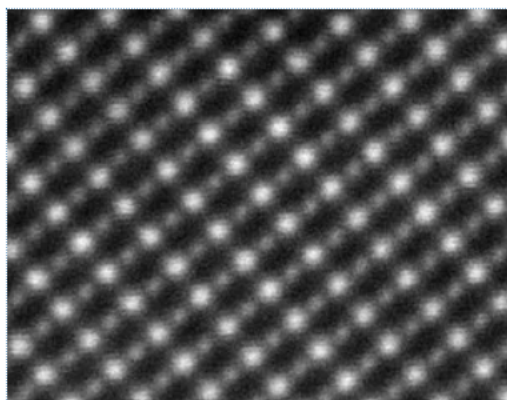
Let us focus on the third figure in the series, where a square plate is divided into four equal parts. If we assume that the area of the plate is one square foot, the scraping of the violin's bow has now created four units of a quarter square foot each, which again total a square foot. Numerically, the number 4 has arisen from the undifferentiated number 1, *but the total area of the plate remains the same*. This means that even though there is a transformation in the quality from undifferentiated to differentiated, *the overall underlying magnitude remains the same*.



This provides a key to a qualitative understanding of Avogadro number when we bring together its different manifestations. Avogadro's number expresses itself sometimes as a whole, and sometimes differentiated, while retaining the same overall magnitude. The differentiated quality in the electrical direction is also seen through an electron microscope where electrical signals are translated into visual form<sup>xxxix</sup>. A comparison of such an image, which is generated at the scale of a few nanometers ( $10^{-9}$  m), with a Chladni plate pattern (of a few centimeters) is given below:



Chladni's plate pattern example



Electron Microscope Image of an oxide

The transformation in the case of Chladni plates is affected through sound and we can actually see the pattern arising. In the case of the Avogadro number, this transformation is more elusive, especially as it is expressed in different ways in a wide variety of phenomena, and only in electrical



phenomena can we detect the discontinuity. But what it *does* lead us to consider is the thought of a fundamental note underlying physical substance, in a musical sense. We encountered this sense of music in the case of chemical combinations, which led John Newlands (1837-1898) to declare<sup>x1</sup>:

*The eighth element, starting from a given one, is a kind of repetition of the first, like the eighth note of an octave in music.*

In case of the Avogadro number, we see this harmonic quality not only in chemical combinations, but in a wide variety of physical phenomena. Due to the crystalline, fixed, and differentiated nature of the Avogadro number in one direction, and an undifferentiated magnitude in the other direction, and also due to the quality of transformation that accompanies its expressions, we can say that they work together like sound and silence. Therefore: **The Avogadro number indicates an inner harmonic quality that pervades physical phenomena.**

This nature of the Avogadro number allows us to perceive the interconnectedness of the entire material world. In a way it expresses a nano-rhythm that pervades the world of substance. In the context of the discussion in section 1, we have worked our way from the “sand hypothesis” of Avogadro’s number through the crystalline, fixed quality into a musical harmonic quality. Hence, in order to keep our ideas in tune with reality, we must not automatically attribute some “thing” to the number, whether that is a ping-pong ball, an atom, or any other entity. Instead, we must stay true to the number itself in both its undifferentiated and differentiated aspects, and see them as expressions of a fundamental note that “sounds” through the physically perceptible phenomena.

This leads to the question: even if this number denotes a fundamental “tone” that is observable in our experiments, how do we gain a relationship to it in terms of an *experience* of this magnitude? How can we comprehend the magnitude of  $6 \times 10^{23}$ ? It is our inability to relate to a size of this magnitude that has caused the sand hypothesis to take root in modern science, as discussed earlier. Having addressed the *quality* of the Avogadro number, how do we now address its *quantity*?

## Quantity of the Avogadro Number

How do we make the proper transition from the quality of the Avogadro number to its quantity? And more importantly, why is the number so big?

Let us focus now on the specific quantity:  $6 \times 10^{23}$ . By stating this magnitude, what exactly are we saying? What we are saying involves the following steps:

1. We have chosen a unit of measure, namely the gram (g).
2. We have decided to express the weights of the chemical elements in terms of this unit (g).
3. We observe the whole number ratios in which chemicals combine, and choose that number of units’ weight for each element.
4. This is 2g for hydrogen gas, 32 g for oxygen gas, 12 g for carbon, 28 g for nitrogen gas, etc.
5. Each of these weights is associated with  $6 \times 10^{23}$  units.

This means that the Avogadro number is dependent on our *choice of unit*. If we had chosen the ounce (oz), whose value is 28.35 g, we would have for the last two steps:

4. This is 2 oz for hydrogen gas, 32 oz for oxygen gas, 12 oz for carbon, etc.
5. Each of these weights is associated with  $1.7 \times 10^{25}$  units.

I am sure none of us have heard the value of  $1.7 \times 10^{25}$  ever referred to as Avogadro's number. This is simply due to the *international scientific consensus for the unit of weight* that is currently used to refer to the Avogadro number. If we had chosen an ounce as our weight unit, the Avogadro number would have been scaled accordingly. This does not mean that the Avogadro number is arbitrary, but it shows how closely it is tied to the unit of weight. If we had chosen a kilogram as the fundamental unit of weight we were going to use for the Avogadro number, its value would have been  $6 \times 10^{26}$ . If we had chosen the pound (lb) as a unit of weight, the Avogadro number would be  $2.7 \times 10^{26}$ . To summarize:

$$\text{Avogadro's number} = 6 \times 10^{23} \text{ (g)} = 1.7 \times 10^{25} \text{ (oz)} = 6 \times 10^{26} \text{ (kg)} = 2.7 \times 10^{26} \text{ (lb)}$$

This variation brings us back to the question addressed in section 2 regarding the *human* measure of different quantities, including weight. A fistful of substance that can be grasped by an adult, such as a pebble, usually weighs about 100 g. A modern-day product designed to be carried in the hand all the time, such as a smartphone, weighs about 150 g. A billiard ball weighs about 180 g. Hence an approximate value for the Avogadro number associated with a weight that “can be grasped” is about  $10^{25}$ – $10^{26}$ .

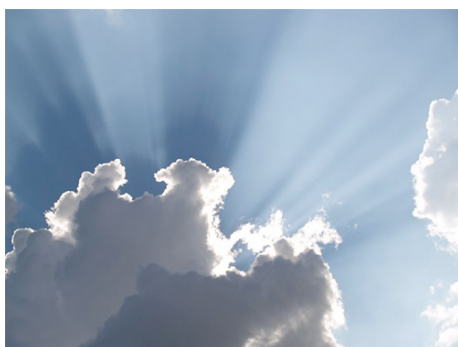
This brings us to the next question, how do we find a way to relate to a number this large? When written out Avogadro's number looks like this: 600,000,000,000,000,000,000,000,000, or in words: six hundred sextillion. It is a truly astronomical number. It is a magnitude we can think in an abstract sense about, but it lies beyond the boundaries of numerical quantities that we can experience directly. It was only with great effort that this number was extracted from the relatively more straightforward experiments regarding size of the electric charge or the conversion of alpha radiation to helium gas. We find ourselves faced with the boundary phenomena discussed in section 4. It could help to cast a glance at a similar question at the other extreme: the question of light speed, or more accurately, the speed at which the effect of light spreads. This is calculated to be about  $3 \times 10^8$  meters per second. When we walk, we travel a couple of meters a second, when running about 5 m/s. The average speed we can throw a ball is about 20 m/s. Hence, light speed is about  $10^7$ – $10^8$  (100,000,000) times faster than any embodied experience of speed. Is there anything within the realm of human experience that could guide us in this respect?

Perhaps we should let the astronomical magnitude of the number guide us and seek the origin of this number in the relation of humankind with the extra-earthly cosmos, i.e. in the stars? The magnitudes involved in changes in the stars involve thousands of years and there is one particular relationship that is deeply related to the human being and has been repeatedly pointed out by the spiritual scientist Rudolf Steiner: the relationship between a single day in the life of a human being and a full cosmic or platonic year (approximately 25,920 years: the time it takes for **one complete cycle of the equinoxes around the ecliptic**). This relationship is particularly special because the number of years in a full platonic year (25,920) is approximately the same as the number of breaths a human takes in a single day (18 breaths a minute  $\times$  60 minutes  $\times$  24 hours). In a sense, the breathing rhythm is a miniature version of the cosmic rhythm expressed on a human scale. What is the ratio between these two scales? In other words, how is a platonic year

related to a single day? There are 9,460,800 (25,920×365) or approximately  $10^7$  days in a cosmic year. Therefore, our experience of a single day is magnified nearly  $10^7$  times when we relate it to the cosmic level. *In other words, the human experience of a cosmic rhythm is nearly  $10^7$  times faster.* This indicates the great difference between the ordinary speeds experienced by us on Earth and light speed.

Please note that the numbers used in the estimates above are not crystalline and exact, since astronomical movements are never exact repetitions. There is always a slight variation in the length of a day, and quite a bit of variation in our breathing rhythm. Thus, these numbers have a living mobile quality to them, and must therefore be handled accordingly.

Let us now examine the relation of light speed to the Avogadro number. In the first place, our experience of beams of light and shadows created by obstacles shows how the activity of light is mainly *linear* or *one-dimensional*. Even though light spreads in all directions three-dimensionally, we are mainly focused on its *activity*, which is called the *rectilinear propagation of light* in the history of physics<sup>xii</sup>. This is the reason why we can see that the edge of a light beam, or the pattern of a shadow cast by an object, is expressed by straight lines.



As we transition into phenomena such as heat propagation in gases and liquids, or heat emission from solids (black-body radiation) that also lead to the determination of the Avogadro number, we move into a realm in which the effects are tangible and three-dimensional. Solids, liquids and gases do not express their presence by the means of straight lines, or even surfaces, but as three-dimensional *volumes*. This also holds true when we electrolyze a mole of substance or observe the decay of a few grams of radium. We can easily grasp a ball or a stone, and we can easily sense water or air by moving our entire hand, which is very different from the way we experience light and shadow. As we transition from the magnitude of light speed to the magnitude of the Avogadro number, we transition from a *one-dimensional* activity to a *three-dimensional* activity of the phenomena under observation. **In other words, the one-dimensional numerical magnitude must be cubed to obtain an estimate of the three-dimensional numerical magnitude.** Sure enough, a relation of  $10^8$  to ordinary speeds becomes  $(10^8)^3$  or  $10^{24}$  for ordinary weights, which is approximately the size of the Avogadro number. Hence, the Avogadro number contains an echo of the relation of the cosmic to the human experience, experienced three-dimensionally. Even modern research is seeking a three-dimensional representation of the Avogadro number, without the scientists realizing the possible origin of their ideas<sup>xiii</sup>.

While this does not enable us to directly experience the Avogadro number in our daily life, tracing the origins of the harmonic quality of the Avogadro number back into a cosmic rhythm establishes a much deeper connection to the world around us. The world is no longer remote and disconnected from us, but we feel ourselves as well as the phenomena around us to be *embedded* in the cosmic whole. This also establishes a harmony between the extra-terrestrial and terrestrial phenomena, putting modern scientific research into a context that humankind can find a place in, once again. This serves as an antidote to the “sand hypothesis” that has penetrated the modern worldview — the “whirling sandstorm of atoms and molecules” ceases. No longer are the stars merely like grains of sand on a beach, but their rhythm is essential to the human rhythm as well as to the physical magnitudes experienced by humans. We can say that:

***The Avogadro number points to a condensed rhythm of the stars in our experience of physical substance.***

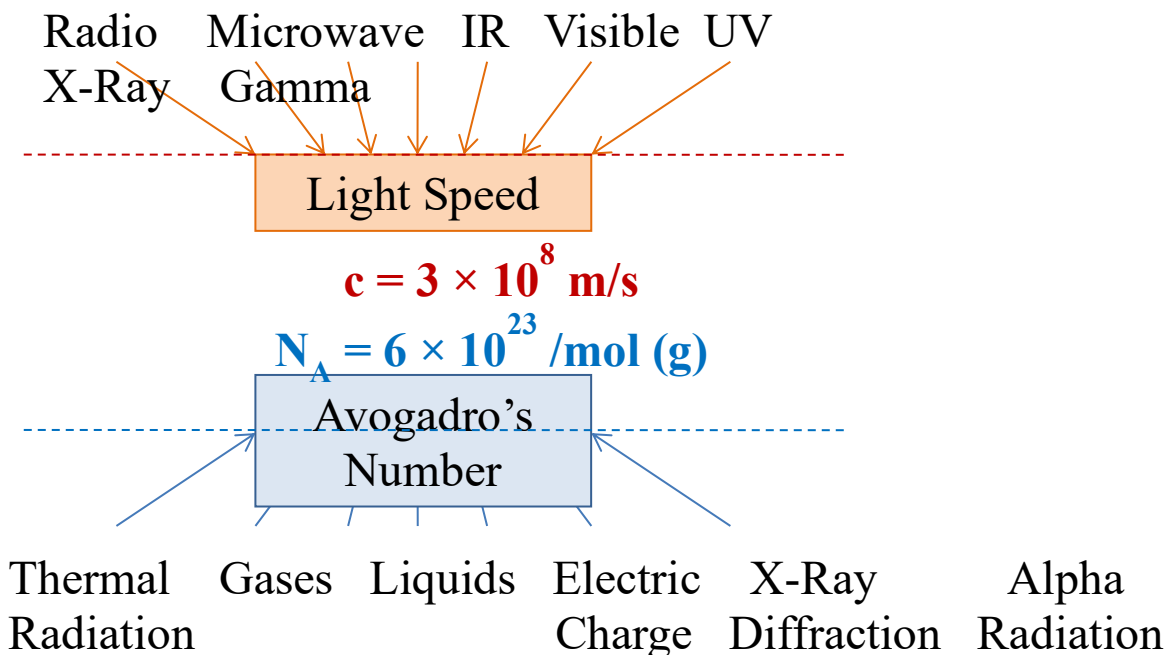
The Avogadro number and the light speed, as magnitudes of the boundary phenomena, are also reintegrated into our experience of the world by offsetting the effects of the microscope and telescope respectively. It is not sufficient for us to passively *observe* the very large and the very small through these instruments, as we need our ideas deepened accordingly to match up to what these instruments reveal to us. We can find our place in the world once again.

### **Further Thoughts on Boundary Phenomena**

Light speed and Avogadro’s number point to the realm of measurements of the very fast and the very small respectively. Both these boundary phenomena open the door to vast realms of modern research. All of modern astronomy implicitly assumes light speed as a basis for estimating cosmic distances, forming the basis for our view of the universe. Focusing on light speed in relation to other speeds experienced by us has opened the door to the Special Theory of Relativity. The relation of light to matter or gravity has opened up the field of General Relativity, while the relation of light speed to electrical phenomena has opened up the field of the so-called electromagnetic radiation. The Avogadro number, on the other hand, leads us into the nano-world through quantum mechanics. Understanding the interpenetration of light and matter has led to the wide field of spectroscopy, which brings both boundary phenomena together. In other words, almost all of twentieth century research has been the elaboration of what has been observed in these two directions. This underlines the importance of exerting the right effort to thoroughly understand these boundary phenomena, even if they are elusive, as this understanding has a cascading impact on nearly a century’s understanding of the physical sciences.

In addition, we must pay particular attention to avoid directly conflating the quantity and the quality, which is very easy to do in case of these boundary phenomena. We have already seen that it is untenable to simply attribute some invisible “things” to the Avogadro number, in effect carrying over the quality of solid matter into the Avogadro number (the sand hypothesis). This direct conflation of quantity and quality has resulted in centuries of confusion, and it is only to avoid this pitfall that we have examined how the quality *of* the quantity guides us in placing the Avogadro number in its right context as a number related to the weight of a substance. We have moved from a direct “number of things” to a “number expressed through what we call things.”

This distinction between qualities and quantities, without disconnecting or conflating them, is an exercise that is also essential at the other boundary: the speed of light. Radiations of different kinds have been conflated into one process since the underlying magnitude — light speed — is common to all of them. It is as inappropriate to conclude that the different qualities of radiation are merely different forms of the same process as it is to conclude that the Avogadro number arises due to different rearrangements of particles in solids, liquids, gases etc. Both these conclusions are habitually taken for granted in modern science, and a real exploration of these fields requires us to go beyond these conflations. The summary of the situation is expressed below:



## Conclusion

The mystery of the Avogadro number has led us into the foundations of physics, since the magnitude of this number has historically made it inaccessible to human experience. In seeking the origins of this number, we had to examine the very nature of numbers as we attribute them to physical phenomena via measurement, and also the wide variety of experiments from which this number itself was drawn. This study led us to see how, as a result of an estrangement from the experience of this number, a deep-seated ping-pong ball/ “sand hypothesis” worldview had sunk its roots into this subject. By extricating ourselves as much as possible from the underlying assumptions regarding this number, and by considering the different experiments that were the basis for deriving the Avogadro number in relation to one another, we begin to see that this number originates in two realms of phenomena: in continuous as well as in discontinuous phenomena. In bringing these two aspects together we can begin to grasp the Avogadro number as a fundamental nano-rhythm of the world around us. By bringing together our experience of the very large and the very small via the magnitudes of light speed and the Avogadro number, we investigated an inner relationship between them. This relationship showed how these magnitudes form an integral whole

in the context of the relationship of the human being to the cosmic rhythm, expressing this rhythm in a modified form both one-dimensionally (as light speed) and three-dimensionally (as Avogadro number in relation to weight). This serves to resolve the world-estrangement that had resulted from the lack of connection to these magnitudes. These identifications not only help us make sense of the large magnitudes involved in these boundary phenomena, but also provide a clue to explore various modern fields of physics by clarifying the relation between quality and quantity at these boundaries.

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<sup>i</sup> Kepler, Johannes, *Concerning the More Certain Fundamentals of Astrology* (1601)

<sup>ii</sup> Gottfried Leibniz Letter to Christian Goldbach, April 17, 1712.

<sup>iv</sup> Milacek, Barbara Roads, *The Microscope and Nineteenth Century Education*, dissertation 1966, p 25

<sup>v</sup> Giovanni Du Pont, 1614.

<sup>vi</sup> <https://www.etymonline.com/word/million>

<sup>vii</sup> <https://www.antidote.info/en/blog/reports/millions-billions-and-other-large-numbers>

<sup>viii</sup> <https://nautil.us/blog/the-word-million-didnt-exist-until-we-needed-it>

<sup>ix</sup> Leewenhoeck A. 1677. Observation, communicated to the publisher by Mr. Antony van Leewenhoeck, in a Dutch letter of the 9 October 1676: concerning little animals by him observed in rain-well-sea and snow water; as also in water wherein pepper had lain infused. *Phil. Trans.* 12, 821–831. (10.1098/rstl.1677.0003)

<sup>x</sup> Carl Sagan American astrophysicist, cosmologist, author and science educator 1934 - 1996

<sup>xi</sup> William Winwood Reade, *The Martyrdom of Man* (1872), Chapter IV, "Intellect"

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<sup>xiii</sup> <https://www.natureinstitute.org/s/GKVijaya-Atomism-an-outline.pdf>

<sup>xiv</sup> <https://www.britannica.com/science/law-of-octaves>

<sup>xv</sup> Jean Perrin, *Brownian Movement and Molecular Reality*, Taylor and Francis: London, 1910 (pg. 91)

<sup>xvi</sup> [https://galileo.phys.virginia.edu/classes/252/kinetic\\_theory.html](https://galileo.phys.virginia.edu/classes/252/kinetic_theory.html)

<sup>xvii</sup> Daniel Bernoulli, *Hydrodynamica, sive de vivibus et motibus fluidorum commentarii, Sectio Decima*, Argentorati 1738, pp. 200-204. An English translation of this section, "On the Properties and Motions of Elastic Fluids, Especially Air" is in Brush (1965), pp. 57-65

<sup>xviii</sup> Buys-Ballot, Christoph. *On the Kind of Motion we Call Heat and Electricity*, *Ann. Phys.* 103: 240. (1858)

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<sup>xx</sup> J. Loschmidt, *Sitzungsber. Kais. Akad. Wiss. Wien, Math. Naturwiss. Kl., II. Abt.* 52, 395 (1866).

<sup>xxi</sup> Jean Perrin, *Brownian Movement and Molecular Reality*, Taylor and Francis: London, 1910 (pg. 7)

<sup>xxii</sup> Jean Perrin, *Brownian Movement and Molecular Reality*, Taylor and Francis: London, 1910

<sup>xxiii</sup> *Ibid.* pg. 21

<sup>xxiv</sup> C. T. Moynihan and H. Goldwhite, *Avogadro's number from the volume of a monolayer*, *J. Chem. Educ.* 1969, 46, 11, 779

<sup>xxv</sup> Thomson, J. J. (1898). *On the Charge of Electricity Carried by the Ions Produced by Röntgen Rays*. *Philosophical Magazine* 46: 528– 545.

<sup>xxvi</sup> Millikan, R. A. (1913). *On the Elementary Electrical Charge and the Avogadro Constant*. *The Physical Review* 2: 109–143.

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<sup>xxix</sup> I. A. Leenson, *Ernest Rutherford, Avogadro's Number, and Chemical Kinetics*, *J. Chem. Educ.* 1998, 75, 8, 998

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<sup>xxxi</sup> George E Smith, and Raghav Seth, *Brownian Motion and Molecular Reality A Study in Theory- Mediated Measurement*, Oxford University Press 2020

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<sup>xxxiv</sup> Ibid. p. 209

<sup>xxxv</sup> Ibid. p. 310

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<sup>xxxvii</sup> <http://elias-gyftopoulos-memorial-collection.unibs.it/EPGyftopoulos-papers/p113-Gyftopoulos-ECOS06-2006.pdf>

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Citation: <https://www.natureinstitute.org/s/gopi-vijaya-avogadros-number.pdf>

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